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Nadal's Limit (L/V) to Wheel Climb and Two Derailment Modes

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Abstract: This paper did a theoretical study on the Nadal's L/V ratio. The analysis is based on a mechanical model of an object sliding on an incline (or slope), which is widely used in college physics. The key is that the direction of frictional forces is always opposite to the direction of the motion of the sliding object. Therefore, there are two directions (upward or downward) for the frictional forces between the object and incline depending on the states of motion of the object. Thus, there must be two L/V ratios for the object sliding on the incline for the same reason. The theoretical demonstration shows that Nadal's L/V is the same with the L/V which governs the downward motion of the object on the incline, because the direction of frictional force between the object and the incline is set to be upwards in the derivation of the Nadal's L/V. Thus, Nadal's L/V is for the object going down the incline. A detail examination was performed on the Nadal's L/V for some typical configurations, such as the critical angle; the zero and 90 degrees angles, further proving that the Nadal's L/V is not for an object going up on the incline, thus cannot be used as the criterion for wheel climb. A new L/V ratio was created by setting the direction of frictional force downwards to simulate the object going up on the incline, and was named as Huang's L/V. Wheel flange/rail contact produces frictional forces between them to consume the pulling power, like a braking to slowdown wheel rotation. Thus, wheel climb is only 1/3 of the whole story of wheel flange/rail contact. The other two are 1). A retarder derailment mode is created by the braking and 2). A braking, large enough, will cause a wheel locked. Therefore, there are two derailment modes with wheel/flange rail contact, wheel climb modes and retarder mode. A method to determine which mode was initiated was demonstrated in the paper. Angle of Attack (AoA) introduces a complicated scenario for wheel climb calculations. It is almost impossible to determine a correct L/V ratio under AoA.

Keywords: Nadal's L/V, Huang's L/V, Friction, Directions of Frictional Forces, Wheel Climb Derailment, Retarder Derailment Mode, Braking and Wheel Locked, Angle of Attack

1. Introduction

Nadal's Limit L/V ratio [4] has been introduced to the railroad industry for a long time, and has been widely used to do wheel derailment analysis. It can be found, in literature, that researches and tests have been done on Nadal's Limit by many scientists and organizations [1, 6, 9, 1-13]. Some tests are claimed to be in agreement with Nadal's Limit L/V ratio. Nadal's Limit is expressed as follows,

$$\frac{L}{V}=\frac{\tan\alpha-\mu}{1+\mu*\tan\alpha} \tag{1}$$

Where L and V are the lateral force and vertical force exerted on truck wheel respectively, α is the flange angle of the wheel, and μ is the friction coefficient between wheel and rail. For some typical wheel flange angles and friction coefficients, Nadal's Limit L/V ratio was computed and tabulated in Table 1.

Based on the L/V ratios from various flange angles and friction coefficients, some researchers and organizations, such as AAR (Association of American Railroads), have adopted L/V=1 (or 0.8) as the maximum value for a railroad truck wheel stability. For a long time, this Nadal's Limit L/V ratio has been used as the criterion to railroad derailment in The United States of America and other countries. In other words, if L/V (lateral force/vertical force) ratio from a truck wheel is larger than Nadal's Limit L/V ratio, the truck will derail. Thus, the design is unacceptable by the criterion. To survive a derailment, truck wheel L/V ratio must be less than Nadal's Limit L/V ratio [3, 5-8, 10, 14, 15].

Table 1. *Nadal's Limit L/V Ratio (computed by Microsoft Excel).*

Friction Coefficient.=	**0.2**	**0.3**	**0.4**	**0.5**
Flange Angle°				
0	-0.2	-0.3	-0.4	-0.5
5	-0.11062	-0.20712	-0.30199	-0.39527
10	-0.02296	-0.11755	-0.20903	-0.29755
15	0.06436	-0.0298	-0.1194	-0.20477
20	0.152663	0.057495	-0.03163	-0.11527
25	0.243356	0.145672	0.055662	-0.02754
30	0.337992	0.236124	0.143806	0.059757
35	0.438393	0.330389	0.234195	0.147974
40	0.546799	0.430264	0.328364	0.238504
45	0.666092	0.537948	0.4281	0.332891
50	0.800141	0.656263	0.535594	0.432938
55	0.954359	0.788984	0.653652	0.540857
60	1.136661	0.941375	0.786023	0.659492
65	1.359202	1.121106	0.937935	0.792647
70	1.641778	1.339907	1.116991	0.945635
75	2.01909	1.616785	1.334811	1.126205
80	2.558204	1.984843	1.6102	1.346226
85	3.408022	2.507494	1.975849	1.624959
90	4.979378	3.323714	2.494238	1.996025

However, many questions arise when one carefully examines the values of Nadal's Limit L/V ratio in Table 1, if Nadal's Limit L/V ratio is to be used as a derailment criterion.

1). At flange angle=0°. By the definition of friction coefficient in mechanical engineering, L/V=friction coefficient. However, Nadal's Limit L/V ratio=- friction coefficient (See Table 1). That is unexplainable if Nadal's Limit is viewed as a derailment maximum value.

2). At flange angle=90°. The support (or constraint) is in the direction of movement. From a mechanical point of view, it is impossible to have a movement (or derailment) no matter how large a force is applied. But Nadal's Limit L/V ratio gives a valid value to show a possible derailment.

3). When the flange angle is small. From mechanical point of view, there is always a possibility for derailment. Nevertheless, Nadal's Limit L/V ratio shows a negative value, which again cannot be explained.

4). Furthermore, Nadal's Limit L/V ratio ranges from negative to positive with increasing flange angles for different friction coefficients. So there is a flange angle at which Nadal's L/V=0. That means any small lateral force (L>0) would cause a derailment. That is not true.

To sum up, does Nadal's Limit L/V ratio really represent the L/V ratio at which the wheel will begin to move to derail? What does Nadal's Limit L/V ratio really mean? Is the lateral force L in Nadal's Limit L/V ratio, the force to push truck wheel over the rail to cause derailment? The objective of this paper is to answer these questions.

A mechanical mathematical model will be established to study the relationship between body movement on slope and external forces applied. The flange angle was modeled as a slope with angle α, while the truck wheel was modeled as a block moving on the slope. From a mechanical point of view, there are two possible moving directions (up and down) for a block on a slope, because there are no constraints in both directions. It can be seen in the following sections, that the two different moving directions will result in two totally different L/V ratios. Thus, the real meaning and the boundary of Nadal's Limit L/V ratio will be understood thoroughly, and consequently a truck wheel derailment criterion can be derived.

2. An Object Moving Down on Slope

As mentioned previously, the study of an object moving on slope is a very useful tool to disclose the secrets inside Nadal's Limit and to understand the mechanism in derailment. For the generality of study, truck wheel-rail system (See Figure 1.) was idealized as a block on slope. See Figure 2 below. Take special note on the block moving direction.

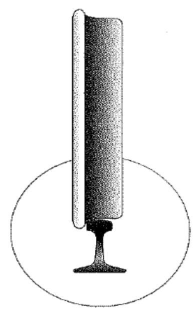

Figure 1. *Truck Wheel-Rail System.*

Equilibrium along the surface of the slope can be established. Sum of forces along the moving direction [2],

$$\sum F=0 \tag{2}$$

$$L\ \cos\alpha - V\ \sin\alpha + \mu\ (L\ \sin\alpha + V\ \cos\alpha)=0 \tag{3}$$

Furthermore,

$$L\ (\cos\alpha + \mu\ \sin\alpha)=V\ (\sin\alpha - \mu\ \cos\alpha) \tag{4}$$

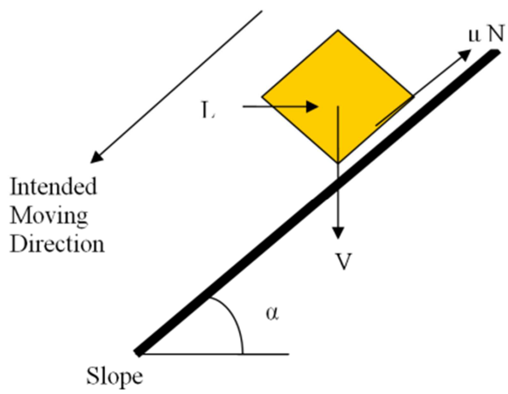

Figure 2. *Truck Wheel Moving Down on Slope.*

Diving V and (Cosα + μ Sinα) on both sides of equation 4, we have an L/V on the right side of the equation. After some mathematical operations, the equation for L/V ratio is obtained. This is exactly the famous Nadal's Limit L/V ratio, Equation (1), as presented previously. After the derivation of the L/V ratio, now it is clear that Nadal's Limit is about an object (truck wheel) moving down on slope. L is the minimum lateral force required to hold the object from sliding down. The corresponding L/V ratio is the minimum ratio for the object not sliding down on that slope. As a result, the questions posed in previous section can now be answered.

At first, the problem of L/V=0 in Nadal's Limit L/V ratio will be discussed. What does it mean by L/V=0? From a mechanical point of view, L/V=0 is impossible for a derailment setting. For illustration purpose, only some typical friction coefficients will be chosen to do the computation. By Nadal's Limit L/V ratio (Equation 1), the results can be shown as follows,

1). When μ=0.2 and α=11.31°, Nadal's Limit L/V=0
2). When μ=0.3 and α=16.71°, Nadal's Limit L/V=0
3). When μ=0.4 and α=21.81°, Nadal's Limit L/V=0
4). When μ=0.5 and α=26.59°, Nadal's Limit L/V=0

In each of these 4 scenarios, condition μ=tan α is satisfied. This condition represents a critical state at which an object will not slide down the slope with L=0.(no lateral force) That is, with a given friction coefficient, there exists an angle (or critical angle) at which the object will not slide down the slope without an applied lateral force L. One can recall that is the general practice in mechanical engineering to obtain the friction coefficient, i.e. μ=tan α.

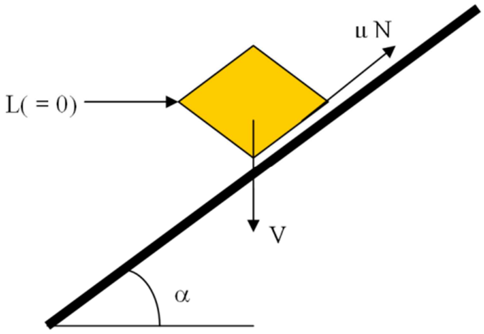

Figure 3. *Critical Angle α --- No sliding down with L=0 (L/V=0).*

It has been demonstrated that, when the angle α is equal to the critical angle, the object will stay on slope by friction only, no lateral force is needed (L=0). Therefore, when the angle α is smaller than the critical angle, the object will certainly stay on slope by friction-support only, no lateral force is needed (L=0). Thus, the negative L/V ratios generated by Nadal's Limit at small flange angles are invalid. The correct L/V values are zero, i.e. L/V=0.

Another question posted in the previous section is about angle α=90°. As mentioned before, movement in lateral force (L) direction, or derailment is impossible. See Figure 4. To prevent the object from moving down on the slope, L/V=1/μ, according to force equilibrium in the vertical direction. L/V values for different friction coefficients are tabulated in Table 2.

Table 2. *L/V Ratio at 90° by force equilibrium.*

Fric. Coeff. μ	**0.2**	**0.3**	**0.4**	**0.5**
α=90°	5.0	3.33333	2.5	2.0

Nadal's Limit L/V ratios at 90° from Table 1 are copied below for comparison.

Table 3. *(α=90° only) Nadal's Limit L/V Ratio.*

Fric. Coeff. μ	**0.2**	**0.3**	**0.4**	**0.5**
α=90°	4.9793	3.3237	2.4942	1.9960

Nadal's Limit L/V ratio should generate the same values as in Table 2. The accuracy in computing Nadal's Limit at α=90° is lost in the process due to the fact that tan α=∞, at α=90°.

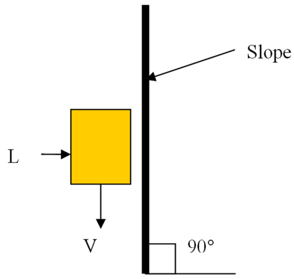

Figure 4. *Slope Angle α=90°.*

It can be seen clearly that Nadal's Limit L/V ratio is for the motion state of an object moving down the slope. However, due to the complexity of motion with frictions, Nadal's Limit is only valid when angle α is larger than the critical angle. That is,

$$\frac{L}{V} = \frac{\tan\alpha - \mu}{1 + \mu * \tan\alpha} \quad \alpha_{critical} < \alpha \leq 90° \tag{5}$$

$$L/V=0 \quad 0° \leq \alpha \leq \alpha_{critical} \tag{6}$$

3. An Object Moving Up on Slope

Nadal's Limit does not indicate the commencement of a derailment because it represents a motion of downward movement. To derail a train, the truck wheel must move upwards to the top of the rail. See Figure 5. Again, the truck wheel was idealized as an object moving up on the slope. An object moving upwards on a slope will be studied.

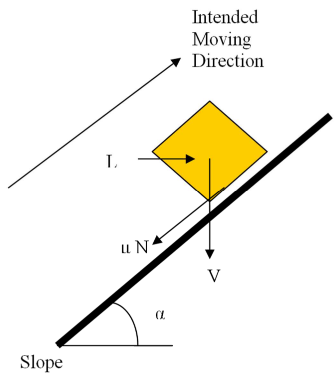

Figure 5. *Truck Wheel Moving Up on Slope.*

Equilibrium along the surface of the slope can be established. Sum of forces along the moving direction,

$$\sum F=0 \tag{7}$$

$$L\ Cos\alpha - V\ Sin\alpha - \mu\ (L\ Sin\alpha + V\ Cos\alpha)=0 \tag{8}$$

Furthermore,

$$L\ (Cos\alpha - \mu\ Sin\alpha)=V\ (Sin\alpha + \mu\ Cos\alpha) \tag{9}$$

Thus we have a new L/V ratio,

$$\frac{L}{V}=\frac{\tan\alpha+\mu}{1-\mu*\tan\alpha} \tag{10}$$

One can realize that this formula is totally different than the Nadal's Limit L/V ratio. In order to distinguish this L/V ratio from the Nadal's Limit L/V ratio, this L/V ratio is named Huang's Limit (named after author's surname). For convenience of discussion, Huang's Limit L/V ratio is computed and tabulated in Table 4.

Table 4. *Huang's Limit L/V Ratio (computed by Microsoft Excel).*

Friction Coef.	0.2	0.3	0.4	0.5
Flange Angle°				
0	0.2	0.3	0.4	0.5
5	0.292561	0.397882	0.505112	0.614303
10	0.389981	0.50282	0.619938	0.741582
15	0.494281	0.617411	0.747927	0.886514
20	0.608005	0.745081	0.893829	1.055805
25	0.734499	0.890547	1.064472	1.259534
30	0.878358	1.060582	1.270144	1.513695
35	1.046155	1.265379	1.52721	1.845403
40	1.247742	1.521137	1.863518	2.304766
45	1.498707	1.855373	2.33077	2.996023
50	1.825371	2.319065	3.03724	4.178052
55	2.276126	3.018659	4.254678	6.720978
60	2.950898	4.220041	6.914719	16.51367
65	4.095014	6.826632	17.71136	-37.388
70	6.515592	17.15433	-32.435	-8.7368
75	15.34698	-34.4805	-8.43208	-4.90332
80	-45.1296	-8.56589	-4.80312	-3.37065
85	-9.1062	-4.8475	-3.32089	-2.5358
90	-5.02079	-3.343	-2.50578	-2.00399

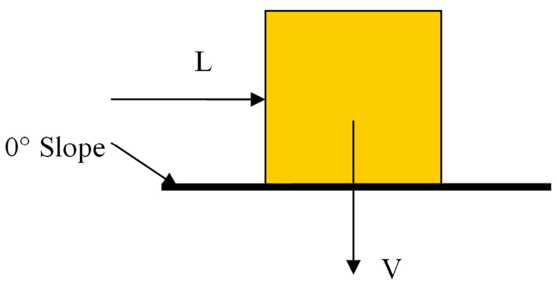

Figure 6. *Sliding on a 0° slope.*

A special case with angle α=0°, can be easily checked to see if Huang's Limit L/V ratio generates the correct results. See Figure 6 for a 0° slope angle scenario. By the definition of friction coefficient, it can be found

$$L/V=\mu \tag{11}$$

Equation (9) is the correct L/V ratio when slope (flange) angle α=0°. Huang's Limit gives exact the same result as Equation (9). Compared with Table 1, Nadal's Limit L/V ratio does not give the correct results.

In Huang's Limit L/V ratio, there also exists a critical angle but at which L/V=∞. Critical angle can be computed by tan α=1/ μ. Critical angles for some typical friction coefficients are tabulated in Table 5.

Table 5. *Critical Angles for Some Typical Friction Coef.*

Friction Coef.	0.2	0.3	0.4	0.5
Critical Angle	78.69°	73.30°	68.20°	63.45°

From a mechanical point of view, L/V=∞ can be understood as the sliding-seizing point for an object on a slope. That means, an object will not be able to move upwards on a slope with critical angle no matter how large the lateral force L. When slope angles are larger than the critical angle, it can be assured that an object will not move upwards under whatever large lateral force L. That explains why there are some negative L/V ratios in Huang's Limit for larger angles, because there will be no upward movement for the object and so the L/V ratios are invalid for angles larger than (and equal to) the critical angle. The significance in this discovery for the railroad industry is that if the wheel-rail contact angle is equal to or larger than the critical angle, there will be no wheel derailment from the L/V ratio mechanism. But reader should note that there is another derailment mechanism besides L/V ratio. In sum,

$$\frac{L}{V}=\frac{\tan\alpha+\mu}{1-\mu*\tan\alpha}\quad 0°\le\alpha<\alpha_{critical} \tag{12}$$

$$L/V=\infty\quad \alpha_{critical}\le\alpha\le 90° \tag{13}$$

4. Angle of Attack and Nadal's Wheel Climb

For a perfect normal wheel-rail setting, axis of wheel (or the center line of axle) will be perpendicular to the line of rail. However, due to curving or some complicated motions of the truck, the axis of wheel will not be perpendicular to the line of rail. This is the scenario in which angle of attack is produced, as illustrated in Figure 7.

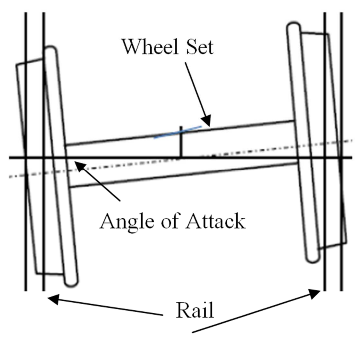

Figure 7. *Angle of Attack (Reference 3).*

Wheel flange will be in contact with rail after an angle of attack is produced. The key point for Nadal's wheel climb is that the wheel will climb with the contact point as a new support of the wheel if L/V ratio is over Nadal's Limit. Question is: is that true? Will that happen in the real services? Further analysis is needed.

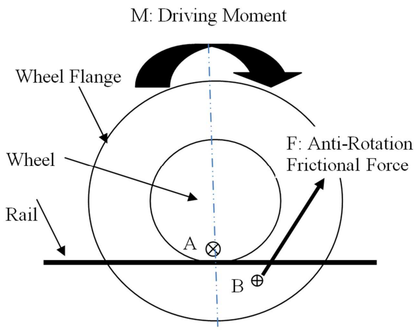

Figure 8. *Wheel-Rail Interaction.*

During the normal operation of the wheel, the wheel has only one contact point (Point A) with the rail. Point A is the support of the wheel and the wheel will rotate around Point A. See Figure 8. When angle of attack is introduced, a point on the wheel flange, say Point B, will come into contact with the rail. What is happening at this instance, from a mechanical point of view, is that a braking effort is applying to the wheel. That is to say, the contact at Point B is like applying a brake to the wheel, and the wheel rotation will be slow down. Anti-rotation frictional force F can be computed from lateral force L and coefficient of friction at Point B. If the anti-rotation moment produced by F is larger than (or equal to) the driving moment M (see Figure 8), the wheel will stop rotating or maybe skid.

It is true that Point B could be used as next pivot point of wheel rotation, if L/V ratio is over Nadal's Limit at Point B. i.e. point B is strong enough to support the wheel to prevent it from moving down. By Nadal's wheel climb theory, this is the wheel climb and derailment occurs. However, this is the scenario which never happens in the real world. If the L/V ratio at Point B is large enough to hold Point B still, the lateral force L will certainly produce a large enough anti-rotation moment (through F) to stop the rotation of the wheel. So the wheel rotation will slow down and finally stop before any Nadal's wheel climb can happen.

When a wheel is rotating on the rail, there is only one rotating point about which the wheel rotates. All other contact points on the wheel can be considered as a retarder. A retarder will slow down the rotation of the wheel or stop it if the braking force is lager enough. It is impossible for a wheel to rotate about a retarder.

From illustrations above, it can be seen clearly that angle of attack will produce a retarder effect to the wheel, causing the wheel to slow down or stop; and that angle of attack will never cause the wheel to move up or climb (The wheel will stop rotating before climb can happen). The base for Nadal's wheel climb is unfounded theoretically and practically, although Nadal's wheel climb theory is very popular currently in the railroad industry.

5. Conclusions

Equations 5, 6, 12 and 13 represent four states of motion for an object on slope. That is, 1). The object will not move down when $\alpha \leq \alpha_{critical}$, 2). The object will move down when $\alpha > \alpha_{critical}$, 3). The object will move up when L increases and 4). The object will not move up when $\alpha \geq \alpha_{critical\ (high\ end)}$.

Angle of attack introduces a retarder to the wheel. This braking effort will reduce the rotation speed of the wheel, and will stop wheel rotation depending on the friction force from braking. Under this circumstance, the wheel may skid but not climb.

Nadal's Limit L/V ratio is the description of the state of a downward movement on slopes. Truck derailment is the upward movement of truck wheel on rails. Therefore, Nadal's Limit L/V ratio is not applicable to the truck derailment problems.

Huang's Limit L/V ratio was derived from the state of upward movement on slopes. It can be used to represent the commencing derailment state of truck wheel. Thus, Huang's Limit is applicable to the truck derailment problems. L/V ratios used in current standards can be largely increased due to that fact that L/V=∞ when flange angle approaches the critical angle, with dynamic factor included.

Values of Huang's Limit L/V ratio increase with increasing of friction coefficients, while values of Nadal's Limit L/V ratio decrease with increasing of friction coefficients. That is why previously some engineering practices applied grease to the rail to prevent derailment. It is clear that applying grease to the rail damage the stability of the truck according to Huang's Limit. However, reducing friction between the wheel flange and the rail will reduce braking effort of the wheel and therefore reduce train power consumption when the wheel is in angle of attack.

Truck derailment is a complicated process. The current truck wheel-rail locking system is a good design to enhance train stability. Besides the L/V ratio, there are some other factors also involved in causing truck derailments. A complete derailment analysis must include all the factors involved. That will be the topic of author's next paper.

Appendix

The paper was meant to publish at a conference in 2008 but was not accepted. Since that time, researches on the subject of wheel climb derailments have been continued. Therefore, research results will be displayed in the addendum for not to change the original paper.

The misleading in Nadal's L/V limit is that directions of frictional forces were not understood and that the force to push up the wheel was confused by many researchers.

Wheel Flange/Rail Contact and Two Derailment Modes

Wheel climb derailments are only 1/3 of the whole process of wheel flange/rail contact derailments. The other two are: 1). Wheel flange/rail contact initiates a braking to the wheel, thus creating a retarder derailment mode to the train. So, there are two derailment modes in the process; and 2). Wheel locked. That is, a braking, if large enough, will lock the wheel. When L/V ≥ 1, wheel locked occurs usually. (Braking theories will not be discussed further here due to out of the scope of the paper.)

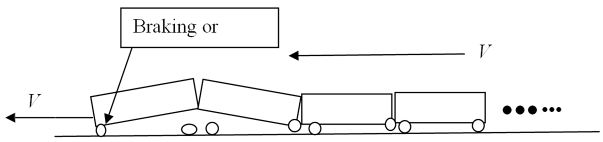

Figure 9. *Retarder Derailment Mode.*

The wheel climb derailment mode is well understood. However, the retarder derailment mode has not been recognized by the railroad industry. There are usually two kinds of the retarder derailment mode. One kind of the retarder mode is shown in Figure 9. How can we tell which derailment mode occurs, retarder mode or wheel climb mode, in a wheel flange/rail contact derailment? L/V values measured, $(L/V)_{measured}$, at the derailment and L/V values calculated, $(L/V)_{calculated}$, from Table 4, Huang's L/V, can be used to determine which mode occurs in the derailment.

$(L/V)_{calculated} > (L/V)_{measured} < 1.0$ Retarder mode

$(L/V)_{calculated} \leq (L/V)_{measured} < 1.0$ Wheel climb mode (most likely)

$(L/V)_{calculated} > (L/V)_{measured} \geq 1.0$ Retarder mode

$(L/V)_{calculated} \leq (L/V)_{measured} \geq 1.0$ Wheel climb mode

Angle of Attack (AoA) and L/V ratios

Wheel flange/rail contact can be either with or without Angle of Attack. We can first look at the problem of flange/rail contact without AoA, as shown in Figure 10. There are two wheel/rail contact points, A and B. Under this circumstance, the wheel section cut view is the standard wheel section view. That means flange angle α can be obtained from standard wheel profile data, thus is easy to obtain to perform L/V ratio calculations.

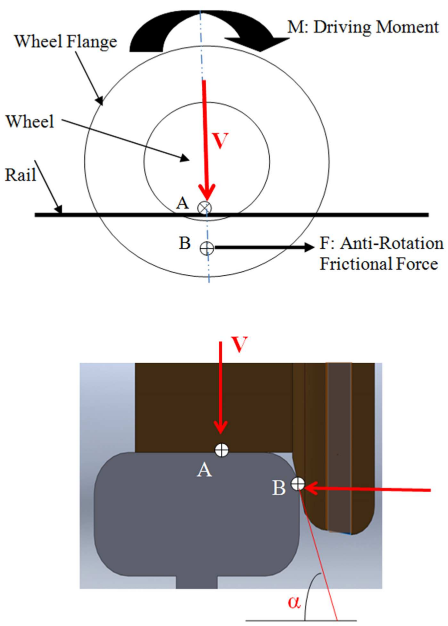

Figure 10. *Wheel Flange/Rail Contact without AoA and Standard Section Cut View.*

However, under the circumstance of flange/rail contact with AoA, as shown in Figure 11, the wheel section cut view at point A, shows that flange angle α is irrelevant; and the wheel section cut view at point B (not shown), indicates that flange angle α cannot be obtained from standard wheel profile data due to AoA. Thus, it is almost impossible to perform L/V ratio calculations under AoA.

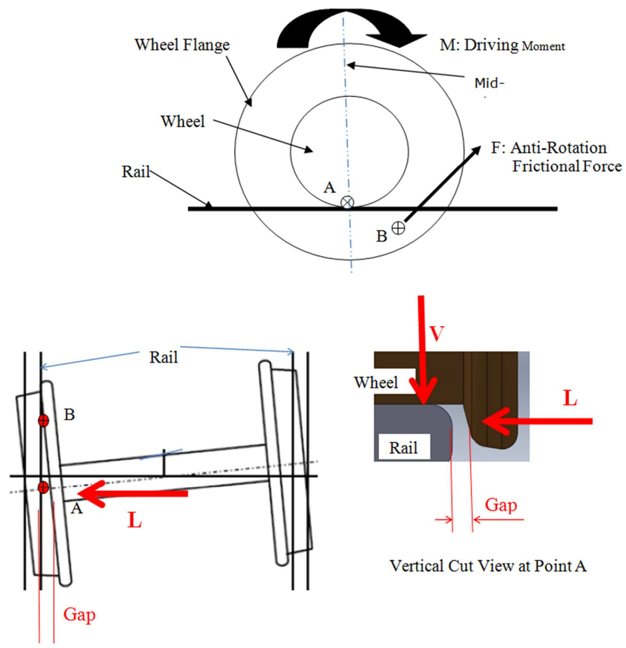

Figure 11. *Wheel Flange/Rail Contact with AoA and Standard Section Cut View.*

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